Spillover Suppression via Eigenstructure Assignment in Large Flexible Structures

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I. Introduction

FOR the active vibration control of large flexible structures, controller designs are usually based on a spatially discrete model that is a finite dimensional approximation of the original system that is inherently a distributed parameter system with infinite dimensions.[1] This approximate model, in spite of being a reduced representation of the original system, is generally of too high an order for a control designer to manage because of limited computing resources. Hence, in many cases, this model must be reduced even further by employing an appropriate model reduction method. As a result, a fundamental problem arises in that an infinite dimensional or high-order system must be controlled by a controller of greatly reduced order. This causes the spillover phenomenon to occur, which degrades control performance and reduces the stability margin. Furthermore, it may destabilize the entire feedback control system.

Balas[2,3] showed that a model reduction in flexible structure systems can result in spillover, which arises from control and measurement coupling between the controlled and residual systems. Furthermore, spillover leads to possible instability in flexible structure systems, which is known as spillover instability.[2–4] In the active control of large flexible structures with discrete sensors and actuators, spillover is unavoidable and undesirable, and, hence, effective spillover reduction or suppression methods are required in control system design.

In this Note, a novel spillover suppression method for large flexible structures is developed by designing an observer that has spillover suppressibility via eigenstructure assignment. This spillover suppression can be achieved by making the observer's left eigenvectors be orthogonal to the column space of an observation spillover term. However, because the observer gain itself enters the observation spillover term, a conventional eigenstructure assignment must be applied in an iterative way to calculate both the observer gain and left eigenvectors simultaneously. To avoid this complexity, the modified eigenstructure assignment is proposed to solve analytically this spillover suppression problem. The effectiveness of the proposed method is verified in simulations where a simply supported flexible beam model having four controlled modes and eight uncontrolled (residual) modes is used.

II. Spillover Instability

Let the controlled and uncontrolled (residual) systems of large flexible structures be represented as follows:

$$\begin{bmatrix}\dot{\boldsymbol{x}}_c(t)\\ \dot{\boldsymbol{x}}_r(t)\end{bmatrix} = \begin{bmatrix}A_c & 0\\ 0 & A_r\end{bmatrix}\begin{bmatrix}\boldsymbol{x}_c(t)\\ \boldsymbol{x}_r(t)\end{bmatrix} + \begin{bmatrix}B_c\\ B_r\end{bmatrix}\boldsymbol{u}(t) \tag{1}$$

where $\boldsymbol{x}_c(t) \in R^{n_c}$ and $\boldsymbol{x}_r(t) \in R^{n_r}$ represent the states of the controlled and residual systems, respectively. By the use of a similarity transformation, the correlations between the controlled system A_c and residual system A_r are converted into the external coupling that is represented by the input matrices B_c and B_r, and the control input $\boldsymbol{u}(t) \in R^m$ is determined depending only on the controlled system. In addition, the system's output $\boldsymbol{y}(t) \in R^l$ can be obtained as follows:

$$\boldsymbol{y}(t) = C_c\boldsymbol{x}_c(t) + C_r\boldsymbol{x}_r(t) \tag{2}$$

Here, it is assumed that (B_c, A_c) is controllable and (C_c, A_c) is observable.

If a state feedback is applied to Eq. (1), the following deterministic estimator (Luenberger observer) must be introduced for any states that are not measured by sensors:

$$\dot{\hat{\boldsymbol{x}}}_c(t) = A_c\hat{\boldsymbol{x}}_c(t) + B_c\boldsymbol{u}(t) + K[\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t)]$$
$$\hat{\boldsymbol{y}}(t) = C_c\hat{\boldsymbol{x}}_c(t) \tag{3}$$

Here, $\hat{\boldsymbol{x}}_c(t) \in R^{n_c}$, $\hat{\boldsymbol{y}}(t)$, and K represent the estimated states, estimated output, and observer gain matrix, respectively. The state feedback control $\boldsymbol{u}(t)$ can then be constructed by using the estimated states as follows:

$$\boldsymbol{u}(t) = -G\hat{\boldsymbol{x}}_c(t) \tag{4}$$

where G is the control gain matrix. If we define the state estimation errors as $\boldsymbol{e}_c(t) = \hat{\boldsymbol{x}}_c(t) - \boldsymbol{x}_c(t)$, then the following equation can be easily derived from Eqs. (1) and (3):

$$\dot{\boldsymbol{e}}_c(t) = (A_c - KC_c)\boldsymbol{e}_c(t) + KC_r\boldsymbol{x}_r(t) \tag{5}$$

Thus, we can construct a composite closed-loop system as follows:

$$\begin{bmatrix}\dot{\boldsymbol{x}}_c(t)\\ \dot{\boldsymbol{e}}_c(t)\\ \dot{\boldsymbol{x}}_r(t)\end{bmatrix} = \left[\begin{array}{cc|c}A_c - B_cG & -B_cG & 0\\ 0 & A_c - KC_c & KC_r\\ \hline -B_rG & -B_rG & A_r\end{array}\right]\begin{bmatrix}\boldsymbol{x}_c(t)\\ \boldsymbol{e}_c(t)\\ \boldsymbol{x}_r(t)\end{bmatrix}$$

$$= \left(\overbrace{\begin{bmatrix}H_{11} & 0\\ H_{21} & H_{22}\end{bmatrix}}^{H_0} + \overbrace{\begin{bmatrix}0 & H_{12}\\ 0 & 0\end{bmatrix}}^{\Delta H}\right)\begin{bmatrix}\boldsymbol{x}_c(t)\\ \boldsymbol{e}_c(t)\\ \boldsymbol{x}_r(t)\end{bmatrix} \tag{6}$$

From Eq. (6), we can see that the residual modes $\boldsymbol{x}_r(t)$ are excited by the controlled modes $\boldsymbol{x}_c(t)$ and the estimation error $\boldsymbol{e}_c(t)$ through the matrix term $-B_rG$, which is called control spillover. The control spillover does not solely destabilize the entire feedback system, but it may degrade the control performance in vibration attenuation. In addition, the estimation process is disturbed by the excitation of residual modes $\boldsymbol{x}_r(t)$ through the matrix term KC_r, which is called observation spillover. This disturbance is originally caused by the sensor outputs being contaminated by the residual modes in Eq. (2).

In Eq. (6), we can always construct a stable closed-loop system by designing the control gain G and observer gain K appropriately, only if the observation spillover is not present ($KC_r = H_{12} = 0$). In this case, the eigenvalues of the composite closed-loop system are exactly those of $A_c - B_cG$, $A_c - KC_c$, and A_r by block triangularity. However, when the observation spillover is present ($KC_r = H_{12} \neq 0$), even small perturbations caused by spillover in the original eigenvalues of H_0 may lead to instability because the stability margin of large flexible structures is especially small, particularly for the eigenvalues of A_r. This phenomenon is known as spillover instability.[2–4]

Spillover tends to reduce the stability margin by shifting the stable eigenvalues of H_0 in Eq. (6) toward the right-half complex plane. The bound on the stability margin reduction due to spillover is given in Ref. 3. Although we should not always expect spillover to produce

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instability, all modal controllers have the potential to generate instability unless the observation spillover is eliminated or suppressed.

III. Spillover Suppression

From Eq. (5), we can obtain the following equation by using modal decomposition ($A_c - KC_c = \Phi\Lambda\Psi^T$):

$$\dot{\boldsymbol{e}}_c(t) = (\Phi\Lambda\Psi^T)\boldsymbol{e}_c(t) + KC_r\boldsymbol{x}_r(t) \tag{7}$$

where $\Lambda \in R^{n_c \times n_c}$ is a diagonal matrix of eigenvalues and Φ and Ψ are the right and left modal matrices, respectively. If the initial values are assumed to be zero, then the solution of Eq. (7) is obtained as

$$\boldsymbol{e}_c(t) = \Phi\int_0^{t_f} e^{\Lambda(t-\tau)}\big(\Psi^T \cdot KC_r\big)\boldsymbol{x}_r(\tau)\,\mathrm{d}\tau, \qquad \boldsymbol{e}_c(0) = 0 \tag{8}$$

Here, to eliminate the effect of the observation spillover completely, we must design an observer that satisfies the following condition:

$$\Psi^T \cdot (KC_r) = [0] \tag{9}$$

which means that all of the left eigenvectors $\boldsymbol{\psi}_i$, $i = 1, 2, \ldots, n_c$, are exactly orthogonal to the subspace spanned by all of the column vectors of KC_r.

Note that, if C_r has a full column rank, the complete spillover elimination as seen in Eq. (9) can be achieved only when the number of independent sensors is equal to or greater than the dimension of the residual system ($l \geq n_r$). However, it is very difficult to satisfy the condition in Eq. (9) because the number of independent sensors is equal to or less than the dimension of the controlled system that is smaller than that of the residual system in a general case.

Hence, based on the concept of modal disturbance suppression,[5,6] we attempt to reduce the detrimental effect of the observation spillover by minimizing the norm of $(\Psi^T \cdot KC_r)$ elementwise. This minimization can actually be achieved by making each eigenvector $\boldsymbol{\psi}_i$, $i = 1, 2, \ldots, n_c$, be as orthogonal as possible to some of the column vectors in KC_r that are selected for their significant contributions to the observation spillover into the ith controlled mode.

On the other hand, the sensitivity of eigenvalues to the perturbations caused by spillover can be minimized, in general, by minimizing the condition number of the corresponding modal matrix. Therefore, we also consider it in this spillover suppression method, and it is well known that near orthogonality of eigenvectors is most preferable to achieve the minimum condition number.[7]

IV. Eigenstructure Assignment for Spillover Suppression

In observer design, it is obvious that our goal for suppressing the observation spillover is to obtain the observer gain K, which makes each left eigenvector satisfy two spillover suppression conditions as stated in the preceding section, by using eigenstructure assignment. However, when a conventional eigenstructure assignment is applied to design such an observer, because the observer gain matrix itself enters the observation spillover term KC_r, a complex and iterative numerical method must be employed to calculate both the observer gain and left eigenvectors simultaneously. To avoid this drawback, the modified eigenstructure assignment that can analytically obtain both the observer gain and left eigenvectors for spillover suppression is proposed in this section.

Let $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_{n_c}\}$ be a self-conjugate set of distinct eigenvalues for simplicity. Then, the left eigenvalue problem for the observer in Eq. (5) can be transformed to the form of the right eigenvalue problem by its transposition as follows:

$$\left\{\boldsymbol{\psi}_i^T\big(\lambda_i I_{n_c} - A_c + KC_c\big)\right\}^T = \big(\lambda_i I_{n_c} - A_c^T + C_c^T K^T\big)\boldsymbol{\psi}_i = 0 \tag{10}$$

where I_{n_c} is an $(n_c \times n_c)$ identity matrix. Hence, by using the duality,[8] we can solve the left eigenstructure assignment for observer by using the right eigenstructure assignment for the controller form $(A_c^T - C_c^T K^T)$ as seen in Eq. (10). Note that, if a system has repeated eigenvalues, the eigenvalue problem can easily be generalized.[9]

First, we define

$$S_{\lambda_i} \equiv \left[\lambda_i I_{n_c} - A_c^T \;\middle|\; C_c^T\right] \tag{11}$$

and a compatibly partitioned matrix as

$$R_{\lambda_i} \equiv \begin{bmatrix} N_{\lambda_i} \\ --- \\ M_{\lambda_i} \end{bmatrix} \tag{12}$$

where the columns of matrix R_{λ_i} form a basis for the null space of S_{λ_i}. If C_c^T has full rank, it can be shown that the columns of N_{λ_i} and M_{λ_i} are linearly independent.[10]

Then, the right eigenvalue problem in Eq. (10) can be rewritten as follows:

$$\left[\lambda_i I_{n_c} - A_c^T \;\middle|\; C_c^T\right]\begin{bmatrix} \boldsymbol{\psi}_i \\ --- \\ K^T\boldsymbol{\psi}_i \end{bmatrix} = 0 \tag{13}$$

Here, we introduce the parameter vector, $\boldsymbol{h}_i \in R^l$, defined by

$$\boldsymbol{h}_i = K^T\boldsymbol{\psi}_i, \qquad i = 1, 2, \ldots, n_c \tag{14}$$

Then, Eq. (13) can be rewritten as

$$\left[\lambda_i I_{n_c} - A_c^T \;\middle|\; C_c^T\right]\begin{bmatrix} \boldsymbol{\psi}_i \\ --- \\ \boldsymbol{h}_i \end{bmatrix} = 0 \tag{15}$$

From Eqs. (11), (12), and (15), there exists a relationship as follows:

$$\begin{bmatrix} \boldsymbol{\psi}_i \\ --- \\ \boldsymbol{h}_i \end{bmatrix} = \begin{bmatrix} N_{\lambda_i} \\ --- \\ M_{\lambda_i} \end{bmatrix} z_i, \qquad i = 1, 2, \ldots, n_c \tag{16}$$

where z_i is the linear combination coefficient vector that can generate both the achievable eigenvectors $\boldsymbol{\psi}_i^a$ from the column space of N_{λ_i} and the achievable parameter vectors $\boldsymbol{h}_i^a$ from the column space of M_{λ_i} simultaneously.

To find the coefficient vectors z_i, we choose the desired parameter vectors $\boldsymbol{h}_i^d$ and the desired eigenvectors $\boldsymbol{\psi}_i^d$ that satisfy the spillover suppression conditions presented in the preceding section and summarized as follows.

Condition 1

The desired parameter vector $\boldsymbol{h}_i^d$ is as orthogonal as possible to some of the column vectors in $C_r = [\boldsymbol{c}_{r1} \;\; \boldsymbol{c}_{r2} \;\; \cdots \;\; \boldsymbol{c}_{rn_r}]$ that are selected for their significant contributions to the observation spillover into the ith controlled mode.

Condition 2

The desired modal matrix $\Psi^d = [\boldsymbol{\psi}_1^d \;\; \boldsymbol{\psi}_2^d \;\; \cdots \;\; \boldsymbol{\psi}_{n_c}^d]$ is orthogonal. By a proper linear combination of the null spaces of each column vector $\boldsymbol{c}_{rj}$, $j = 1, 2, \ldots, n_r$, with a weighting factor $\boldsymbol{\alpha}_i$, we can obtain such desired parameter vectors $\boldsymbol{h}_i^d$ as in condition 1 from the following equation:

$$\boldsymbol{h}_i^d = \left[\ker\!\big(\boldsymbol{c}_{r1}^T\big) \;\middle|\; \ker\!\big(\boldsymbol{c}_{r2}^T\big) \;\middle|\; \cdots \;\middle|\; \ker\!\big(\boldsymbol{c}_{rn_r}^T\big)\right] \cdot \boldsymbol{\alpha}_i \qquad i = 1, 2, \ldots, n_c \tag{17}$$

where $\ker(\cdot)$ represents the null space of $(\cdot)$. Here, $\boldsymbol{\alpha}_i$ can be determined to give more weight on the corresponding null spaces of some column vectors that play a significant role in the observation spillover into the ith controlled mode than others. Of course, it is based on an assumption that we have the full knowledge of which residual modes have more significant influence of spillover on the

ith controlled mode than others. Note that there is no need for n_c desired parameter vectors to be independent.

When the desired parameter vector $\boldsymbol{h}_i^d$ is obtained to satisfy condition 1, then there may be many eigenvectors that satisfy Eq. (14). Thus, by using the redundant freedom in this set of eigenvectors, we can select the desired eigenvectors to satisfy condition 2. Note that, from Eq. (17), the desired parameter vector $\boldsymbol{h}_i^d$ is obtained to be real because the residual output matrix C_r is a real one. Hence, it is more desirable that the desired eigenvalues and eigenvectors should be real rather than complex.

Once the desired parameter vector $\boldsymbol{h}_i^d$ and desired eigenvector $\boldsymbol{\psi}_i^d$ that satisfy the spillover suppression conditions are selected, then the coefficient vector $\boldsymbol{z}_i$ can be obtained by the following equation:

$$\boldsymbol{z}_i = \begin{bmatrix} N_{\lambda_i} \\ --- \\ M_{\lambda_i} \end{bmatrix}^{\dagger} \begin{bmatrix} \boldsymbol{\psi}_i^d \\ --- \\ \boldsymbol{h}_i^d \end{bmatrix} \tag{18}$$

where the brackets and the dagger superscript denotes the pseudoinverse of a given matrix in brackets. Although the desired eigenvector $\boldsymbol{\psi}_i^d$ does not reside exactly in the achievable eigenspace, the achievable eigenvector $\boldsymbol{\psi}_i^a$ can be obtained in the least-square sense by using $\boldsymbol{z}_i$ from Eq. (18) as follows:

$$\boldsymbol{\psi}_i^a = N_{\lambda_i}\boldsymbol{z}_i \tag{19}$$

In the same way, the achievable parameter matrix $\boldsymbol{h}_i^a$ can also be obtained by using $\boldsymbol{z}_i$ as follows:

$$\boldsymbol{h}_i^a = M_{\lambda_i}\boldsymbol{z}_i \tag{20}$$

Then, the observer gain K can be obtained from Eq. (14) and given by the matrix equation as follows:

$$K^T = H^a(\Psi^a)^{-1} \tag{21}$$

where $H^a = [\boldsymbol{h}_1^a \;\; \boldsymbol{h}_2^a \;\; \cdots \;\; \boldsymbol{h}_{nc}^a]$.

V. Numerical Example

In numerical experiments, a simply supported flexible beam model was used to verify the validity and effectiveness of the proposed spillover suppression method. The beam was modeled to have four controlled modes that were controlled by four collocated actuators and sensors and eight residual modes that were to generate spillover as follows:

Table 1 Eigenvalues of open loop, case 1, and case 2

Open loop	Case 1	Case 2
Controlled system	−6.2974	−4.8226 + 1.9100i
−0.0006 ± 0.3679i	−4.0445	−4.8226 − 1.9100i
−0.0005 ± 0.2807i	0.0499 + 1.5609i	−0.0541 + 1.3581i
	0.0499 − 1.5609i	−0.0541 − 1.3581i
Residual system	−1.2943 ± 0.9214i	−1.5397 ± 0.9174i
−0.0005 ± 0.1860i	−0.2108	−0.2034
−0.0005 ± 0.1058i	−0.0087 ± 0.0495i	−0.0096 ± 0.1969i
−0.0005 ± 0.0472i	−0.0318 ± 0.1704i	−0.0319 ± 0.1720i
−0.0005 ± 0.0118i	−0.0053	−0.0054
	−0.0269 ± 0.0309i	−0.0256 ± 0.0309i
	−0.0113 ± 0.1993i	−0.0141 ± 0.0494i

The eigenvalues of the open-loop system are densely spaced as described in Table 1, which is one of the representative characteristics of large flexible structures. Let the desired eigenvalues for controller be

$$\Lambda_{\text{controller}} = \{-0.4 \quad -0.5 \quad -0.8 \quad -1.0\}$$

and an eigenstructure assignment scheme is applied in controller design.

In these simulations, we consider two cases of observer design: One is that a conventional eigenstructure assignment is applied to minimize the sensitivity of eigenvalues without any consideration for spillover suppression, and the other is that the proposed eigenstructure assignment for spillover suppression is applied. Note that, in these two cases, the identical controller is applied. Let the desired eigenvalues of observer be the same for the two cases as

$$\Lambda_{\text{observer}} = \{-2.2 \quad -2.5 \quad -2.8 \quad -3.0\}$$

In the first case of observer design (case 1), we try to minimize only the sensitivity of eigenvalues; hence, the minimum condition number of the achieved left modal matrix $k(\Psi^a)$ is obtained as 263.5680. Here, $k(cdot)$ represents the condition number of $(\cdot)$. Then, the observer gain matrix K and its Frobenious norm are calculated as follows:

$$K = \begin{bmatrix} -64.4777 & 29.5985 & 26.1795 & -61.2951 \\ -29.9610 & 30.6381 & -29.7092 & 23.1537 \\ 4.0863 & -1.8461 & -1.7412 & 3.9883 \\ 2.8071 & -3.1451 & -3.1451 & -2.4602 \end{bmatrix}$$

$$\|K\|_{\text{fro}} = 113.1525$$

$$\dot{\boldsymbol{x}}_c(t) = A_c\boldsymbol{x}_c(t) + B_c\boldsymbol{u}(t) = \left[\begin{array}{cc|cc} \multicolumn{2}{c|}{[0]_{2\times 2}} & \multicolumn{2}{c}{I_{2\times 2}} \\ \hline -0.0788 & 0 & -0.0011 & 0 \\ 0 & -0.1353 & 0 & -0.0011 \end{array}\right]\boldsymbol{x}_c(t) + \left[\begin{array}{cccc} \multicolumn{4}{c}{[0]_{2\times 4}} \\ \hline 0.5211 & -0.2319 & -0.2319 & 0.5211 \\ 0.4179 & -0.5211 & 0.5211 & -0.4179 \end{array}\right]\boldsymbol{u}(t)$$

$$\dot{\boldsymbol{x}}_r(t) = A_r\boldsymbol{x}_r(t) + B_r\boldsymbol{u}(t) = \left[\begin{array}{cccc|cccc} \multicolumn{4}{c|}{[0]_{4\times 4}} & \multicolumn{4}{c}{I_{4\times 4}} \\ \hline -0.0001 & 0 & 0 & 0 & -0.001 & 0 & 0 & 0 \\ 0 & -0.0022 & 0 & 0 & 0 & -0.001 & 0 & 0 \\ 0 & 0 & -0.0112 & 0 & 0 & 0 & -0.001 & 0 \\ 0 & 0 & 0 & -0.0346 & 0 & 0 & 0 & -0.001 \end{array}\right]\boldsymbol{x}_r(t)$$

$$+ \left[\begin{array}{cccc} \multicolumn{4}{c}{[0]_{4\times 4}} \\ \hline -0.4179 & -0.5211 & -0.5211 & -0.4179 \\ -0.5211 & -0.2319 & 0.2319 & 0.5211 \\ 0.2319 & -0.4179 & -0.4179 & 0.2319 \\ -0.2319 & -0.4179 & 0.4179 & 0.2319 \end{array}\right]\boldsymbol{u}(t)$$

$$\boldsymbol{y}(t) = C_c\boldsymbol{x}_c(t) + C_r\boldsymbol{x}_r(t) = \left[\begin{array}{c|cc} & 0.5211 & 0.4179 \\ [0]_{4\times 2} & -0.2319 & -0.5211 \\ & -0.2319 & 0.5211 \\ & 0.5211 & -0.4179 \end{array}\right]\boldsymbol{x}_c(t) + \left[\begin{array}{c|cccc} & -0.4179 & -0.5211 & 0.2319 & -0.2319 \\ [0]_{4\times 2} & -0.5211 & -0.2319 & -0.4179 & -0.4179 \\ & -0.5211 & 0.2319 & -0.4179 & 0.4179 \\ & -0.4179 & 0.5211 & 0.2319 & 0.2319 \end{array}\right]\boldsymbol{x}_r(t)$$

Table 2 Summarized results of case 1 and case 2

Case	$k(\Psi^a)$	$\|K\|_{\text{fro}}$	$\|KC_r\|_{\text{fro}}$	$\|\Psi^T(KC_r)\|_{\text{fro}}$
1	263.5680	113.1525	60.7262	3.0335
2	2801.7103	113.0615	47.0881	2.6789

As a result, the norm of the observation spillover term $\|KC_r\|_{\text{fro}}$ is 60.7262. In this case, spillover instability occurs, which can be easily verified from the eigenvalues of the composite system in Eq. (6) as described in Table 1 (case 1). The underlined eigenvalues in case 1, which are shifted by spillover, makes the entire feedback control system unstable.

Next, in the second case (case 2), the achievable parameter matrix H^a is obtained as follows:

$$H^a = \begin{bmatrix} -10.7076 & -20.5166 & -11.0363 & 13.5645 \\ 7.9923 & 14.7979 & 9.9422 & -12.0749 \\ -2.0429 & -2.8253 & -5.7019 & 6.7023 \\ -2.6599 & -6.3837 & 1.5093 & -1.4936 \end{bmatrix}$$

In addition, the condition number of the achievable modal matrix $k(\Psi^a)$ is 2081.7103. Then, the observer gain matrix K and its Frobenious norm are calculated as follows:

$$K = \begin{bmatrix} -11.5548 & -13.9220 & 45.3413 & -59.0716 \\ -49.3056 & 50.2492 & -37.7653 & 21.2715 \\ 2.5023 & -0.5594 & -2.2824 & 3.8838 \\ 3.7553 & -4.1166 & 3.4873 & -2.3422 \end{bmatrix}$$

$$\|K\|_{\text{fro}} = 113.0615$$

As a result, the norm of the observation spillover term $\|KC_r\|_{\text{fro}}$ is 47.0881. In this case, the stable entire feedback control system is obtained by suppressing the observation spillover, which can be seen from the eigenvalues of the composite system in Table 1 (case 2).

In simulations, two cases of observer design are compared in almost the same norm condition of K as seen in Table 2. In case 1, we achieve a minimum condition number of Ψ^a for the preassigned eigenvalues, but it can not avoid spillover instability. On the other hand, in case 2, the condition number $k(\Psi^a)$ increases compared with case 1 because some part of the design freedom for eigenvectors is used to design the parameter vectors to satisfy condition 1. However, we can reduce the norm of the observation spillover term KC_r, and spillover instability in case 1 is recovered. Consequently, the observer design in case 2 has shown to be more effective and successful in spillover suppression than that of case 1 in which the spillover suppression is dealt with simply as a robust stability problem.

VI. Conclusions

This Note presents a new spillover suppression method using eigenstructure assignment, which can prevent possible instability in the active control of large flexible structures caused by the neglected dynamics of the higher modes of vibration by reducing their detrimental effect of spillover. In simulations, by the comparison of two cases of observer design, the proposed method has been proven to be successful in the spillover suppression of large flexible structures.

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Solar Sail Hybrid Trajectory Optimization for Non-Keplerian Orbit Transfers

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Introduction

SOLAR sails have long been seen as an attractive concept for low-thrust propulsion. They transcend reliance on reaction mass and have the ability to provide a small, but continuous, acceleration. Because propellant mass is not an issue, high-performance sails can enable new exotic non-Keplerian orbits (NKOs)[1] that are not feasible for conventional chemical or electric propulsion. A constant out-of-plane sail force is utilized to raise the spacecraft's orbit high above the ecliptic plane in two- or three-body systems. Potential benefits to the science community are large. Circular, displaced orbits can be used to provide continuous observation of the solar poles or to provide a unique vantage point for infrared astronomy. (There is much less resolution-limiting dust out of the ecliptic plane enabling smaller telescope mirror dimensions for equivalent performance.) Very-high-performance sails can even levitate, in equilibrium, at any point in space.

Three parameters of the NKO determine what sail acceleration and orientation is required: the vertical displacement z above the ecliptic plane in the direction of the ecliptic normal, the horizontal distance ρ from the sun in the ecliptic plane, and the orbit period T, which is usually taken to be one year so that the sail orbits synchronously with the Earth. The sail lightness number β is defined as the ratio of the sail characteristic acceleration [solar radiation pressure induced acceleration at 1 astronomical unit (AU)] to the solar gravitational acceleration at 1 AU. Because both have an inverse square form, the sail lightness number is a dimensionless constant.

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